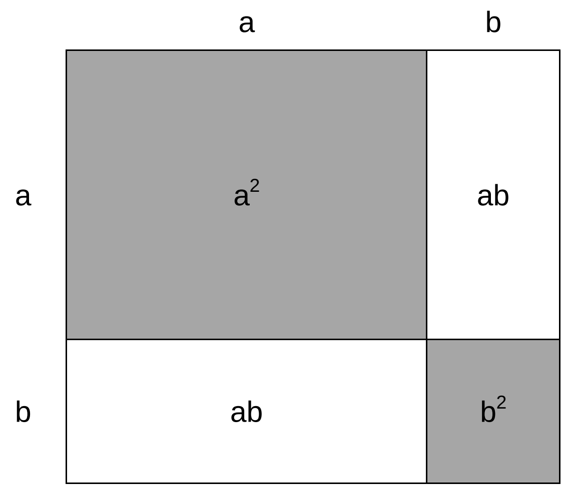
In particular, the binomial theorem, $(a + b)^2 = a^2 + 2ab + b^2$, which greatly simplifies a lot of mental arithmetic, allowing you to amaze your friends and confound your enemies.

(In the spirit of a previous ~~rant~~ discussion, I include a geometric perspective on the binomial theorem to supplement the algebraic one:



As a simple example, what is 81 squared? Looks tough, unless you apply the binomial theorem, and calculate it as $(80 + 1)^2$, in which case no problem. $80^2 + 2(1)(80) + 1^2 = 6400 + 160 + 1 = 6561$. Simple distributivity is also invaluable. In college a girlfriend was filling out a housing form and needed to calculate 60% of $147. Easy: (0.50 + 0.1)* $147 = $73.5 + $14.7 = $88.2. When I (absent-mindedly and rather stupidly) blurted that out, she scoffed, took out pencil and paper, did the calculation, then looked up at me as though I were in league with the devil. But the calculation is actually straightforward. It's especially easy when the numbers are evenly distributed about a multiple of ten. For example, 87*93 looks nasty, but when calculated as $(90 - 3)(90 + 3) = 90^2 - 3(90) + 3(90) - 3^2 = 8100 - 9 = 8091$, there's nothing to it.[i]

I'd also include learning the inverses of the numbers 1 through 12. Sometimes rather than dividing by a number it's easier to multiply by the inverse (or *vice versa*). Divide 93 by 7? Easier to multiply 93 by 0.14 (two decimal places is enough for many purposes); (0.1 + 0.04)*93 = 9.3 + (0.04)*(90 + 3) = 9.3 + 3.6 + 0.12 = 12.9 + 0.12 = 13.02. Most of the inverses are easy to remember; that of 7 is one of the more obscure ones, but can be remembered because 7*14 = 98 (i.e., almost 100). To get more precision, if needed, is also easy: 98 is 2 short of 100, so 7*(14 2/7) = 100, and 2/7 = 2*(0.14) = 0.28, so 1/7 to four decimal places is 0.1428.[ii]

Last, I regret the passing of the slide rule. Not just because I'm a geezer, because that would never happen, but because there's nothing like calculating on a slide rule to place a premium on mental arithmetic. Did you, in the heat of time pressure on an exam, inadvertently look at the CI scale, instead of the C scale, or *vice versa*? If you roughed out the calculation mentally, you'll instantly recognize your mistake. Pretty soon doing mental arithmetic becomes second-nature.

As an aside, the point of these math topics, indeed, of studying mathematics itself, is not the matter at hand itself. Few people will ever solve line integrals - or even quadratics - at work in their careers. It is, instead, to learn how to think, how to approach a given problem. Is there another, simpler, way to solve the same problem? Or more generally, what is the *best* way? That's where to approach a problem. What is the problem, in its essence, and what is the simplest way of approaching it (rather than simply plowing ahead with grim determination)?

Mathematics, for most people, merely provides a proving ground that poses such questions, the gym in which to work out in preparation for future challenges, much like doing squats to prepare for the high jump. It is to the place in which to learn to extract the abstract aspects of a given problem that transcend the details of a specific case, which IMO is the apotheosis of intellectual development.

And therein lies its value.

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[i] Even if they're not symmetrically distributed about a multiple of ten, it's still pretty easy; it's just that the cross terms won't entirely vanish.

[ii] Actually, to get more precision while avoiding multiplying by a four-digit number, it's much easier to note that since 93*0.***14*** = 13.02, 93*0.14***28*** would just add 2(13.02)(0.01) giving 13.02 + 0.2602, or rounding off, 13.28.